

# ***crab cavities for the LHC Upgrade***

Frank Zimmermann

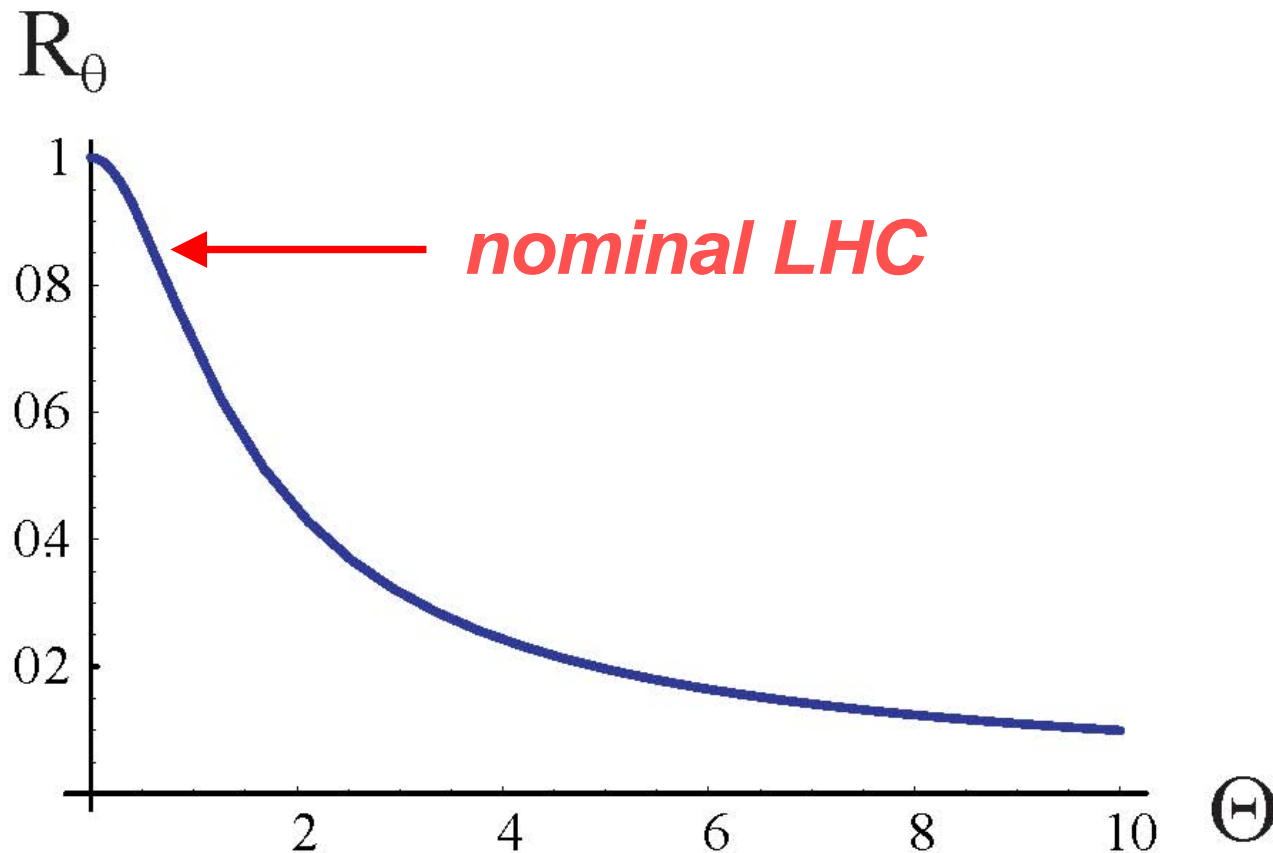
Thanks to  
Kazunori Akai, John Byrd, Kazuhito Ohmi,  
Katsunobu Oide, Francesco Ruggiero,  
Joachim Tuckmantel, Tanaji Sen

# (1) need for beam-beam compensation

- **nominal LHC parameters are challenging & “at the edge”:**
  - ❖ ~20% geometric luminosity loss from crossing angle
  - ❖ chaotic particle trajectories at  $4\text{-}6\sigma$  due to long-range beam-beam effects
- **if we increase #bunches or bunch charge, or reduce  $\beta^*$ :**
  - ❖ long-range beam-beam effects require larger crossing angle
  - ❖ but geometric luminosity loss would be unacceptable!

$$R_{\theta} = \frac{1}{\sqrt{1 + \Theta^2}}; \quad \Theta \equiv \frac{\theta_c \sigma_z}{2\sigma_x} \quad \text{Piwinski angle}$$

luminosity reduction factor

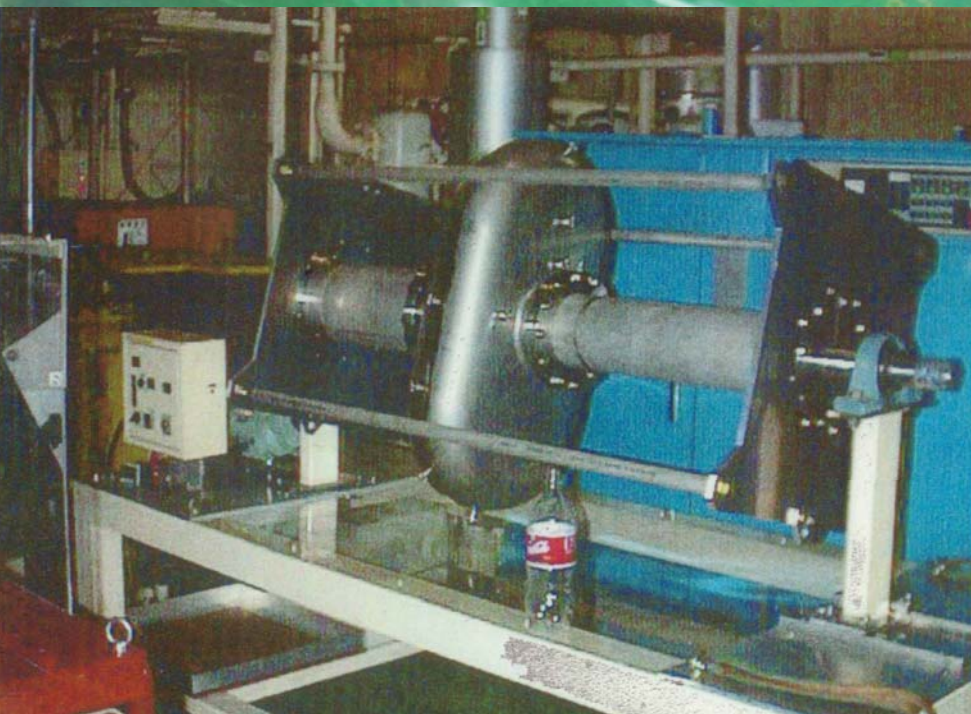


**to boost LHC performance further various approaches have been proposed:**

- 1) **increase crossing angle AND reduce bunch length**  
(higher-frequency rf & reduced longitudinal emittance)  
[J. Gareyte; J. Tuckmantel, HHH-20004]
- 2) **reduce crossing angle & apply “wire” compensation**  
[J.-P. Koutchouk]
- 3) **crab cavities → large crossing angles w/o luminosity loss**  
[R. Palmer, 1988; K.~Oide, K. Yokoya, 1989; KEKB 2006]
- 4) **collide long intense bunches with large crossing angle**  
[F. Ruggiero, F. Zimmermann, ~2002]

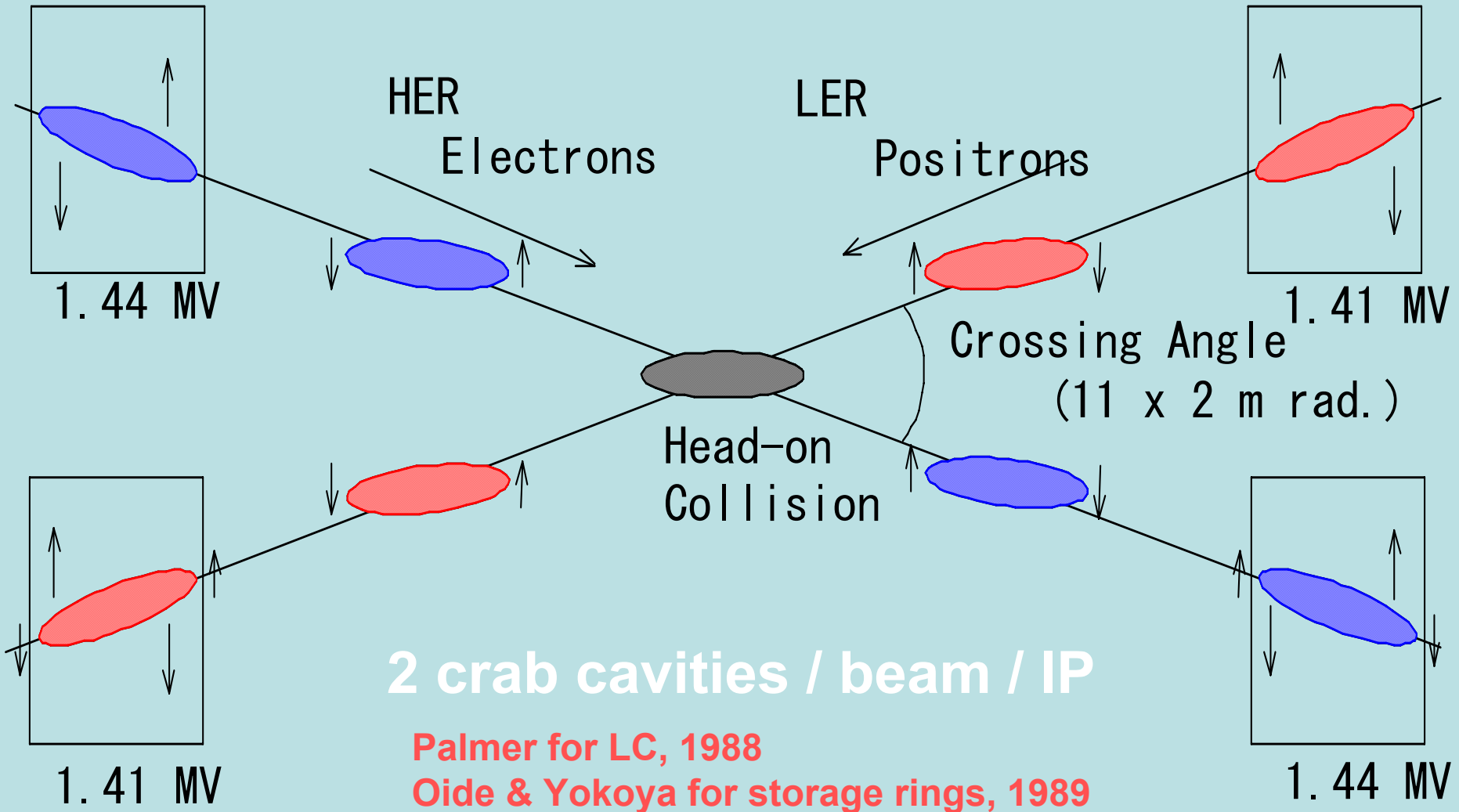


s.c. crab cavity production  
at KEKB



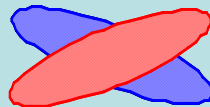
# Super-KEKB crab cavity scheme

RF Deflector  
( Crab Cavity )



Palmer for LC, 1988

Oide & Yokoya for storage rings, 1989



first crab cavities will be installed  
at KEKB in early 2006

# history of s.c. crab cavity developments

- ❖ CERN/Karlsruhe sc deflecting cavity for separating the kaon beam, 1970's, 2.86 GHz\*
- ❖ Cornell 1.5 GHz crab cavity 1/3 scale models 1991\*
- ❖ KEK 500 MHz crab cavity with extreme polarization, 1993-present, for 1-2 A current, 5-7 mm bunch length
- ❖ FNAL CKM deflecting cavity, 2000-present\*
- ❖ KEK 2003 new crab cavity design for Super-KEKB, 10 A beam current, 3 mm bunch length, more heavily damped (coaxial & waveguide)
- ❖ Daresbury is studying crab cavities for ILC, 2005
- ❖ Cornell and LBNL are interested in developing crab cavities for Super-LHC

\*H. Padamsee, Daresbury Crab Cavity Meeting, April 2004



bunch shortening rf voltage:

$$V_{rf} \approx \left[ \frac{\varepsilon_{\parallel, rms}^2 c^3 C \eta}{E_0 2\pi f_{rf}} \right] \frac{1}{\sigma_z^4} \approx \left[ \frac{\varepsilon_{\parallel, rms}^2 c^3 C \eta}{E_0 2\pi f_{rf}} \right] \frac{\theta_c^4}{0.7^4 16 \sigma_x^{*4}}$$

unfavorable scaling as 4<sup>th</sup> power of crossing angle and inverse 4<sup>th</sup> power of IP beam size; can be decreased by reducing the longitudinal emittance; inversely proportional to rf frequency

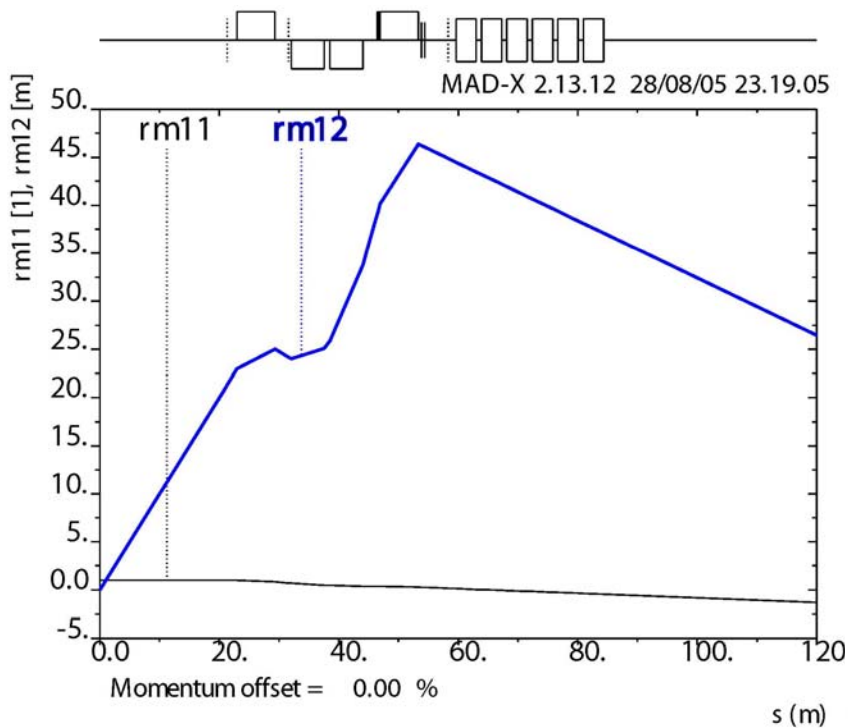
crab cavity rf voltage:

$$V_{crab} = \frac{cE_0 \tan(\theta_c / 2)}{e2\pi f_{rf} R_{12}} \approx \frac{cE_0}{e4\pi f_{rf} R_{12}} \theta_c$$

proportional to crossing angle & independent of IP beam size; scales with 1/R<sub>12</sub>; also inversely proportional to rf frequency



# R12 & R22(R11) from MAD

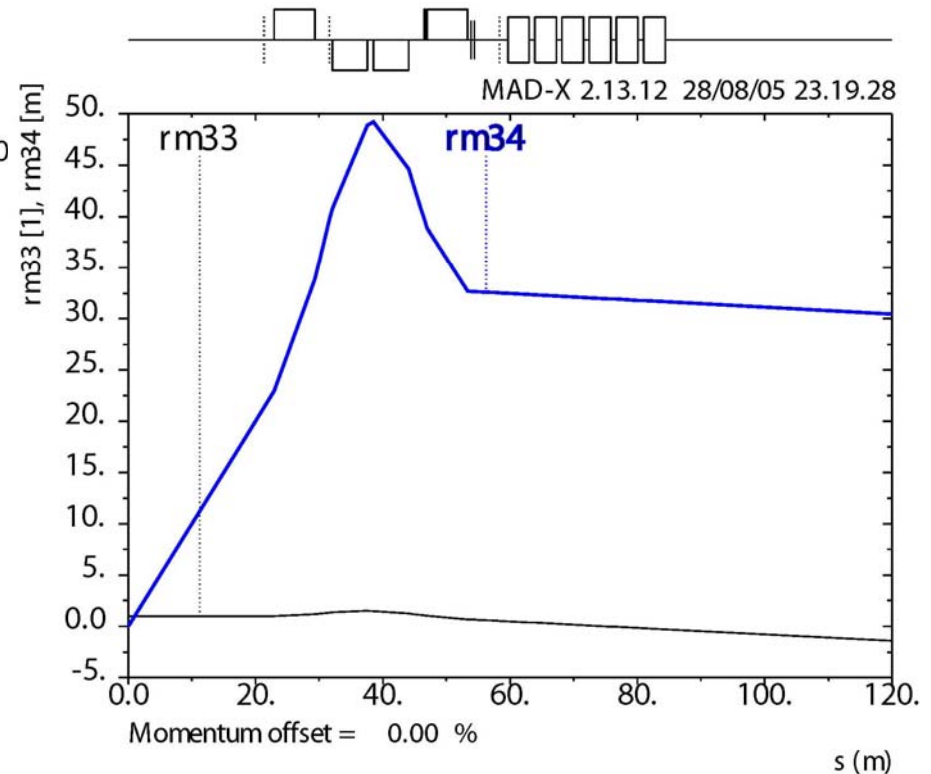


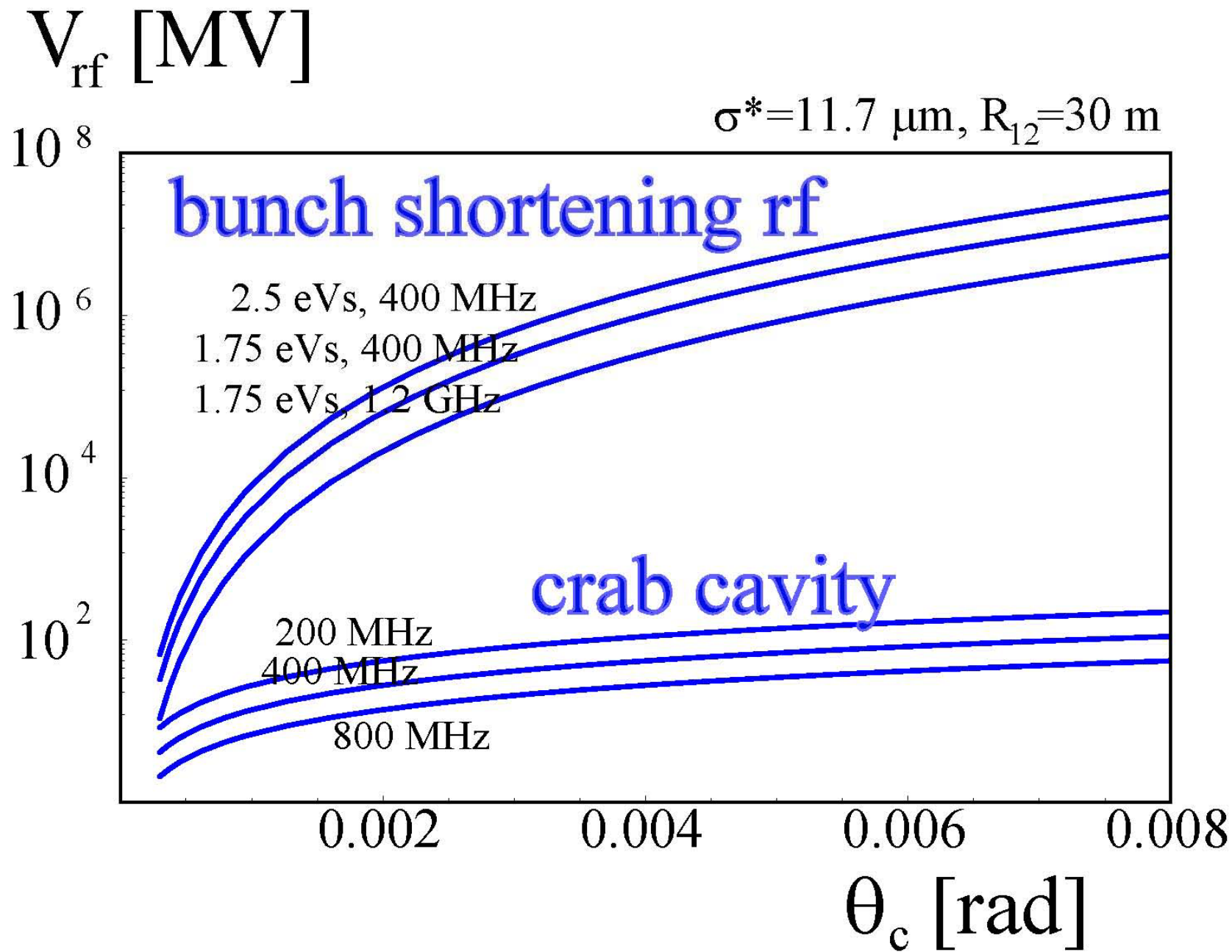
*nominal LHC optics*

$$|R_{12,34}| \sim 30-45 \text{ m}$$

$$|R_{22,44}| \sim 1$$

(from crab cavity to IP)





$V_{\text{rf}}$  [MV]

$\sigma^* = 11.7 \mu\text{m}$ ,  $R_{12} = 30 \text{ m}$

bunch shortening rf

200 MHz

400 MHz

800 MHz

crab cavity

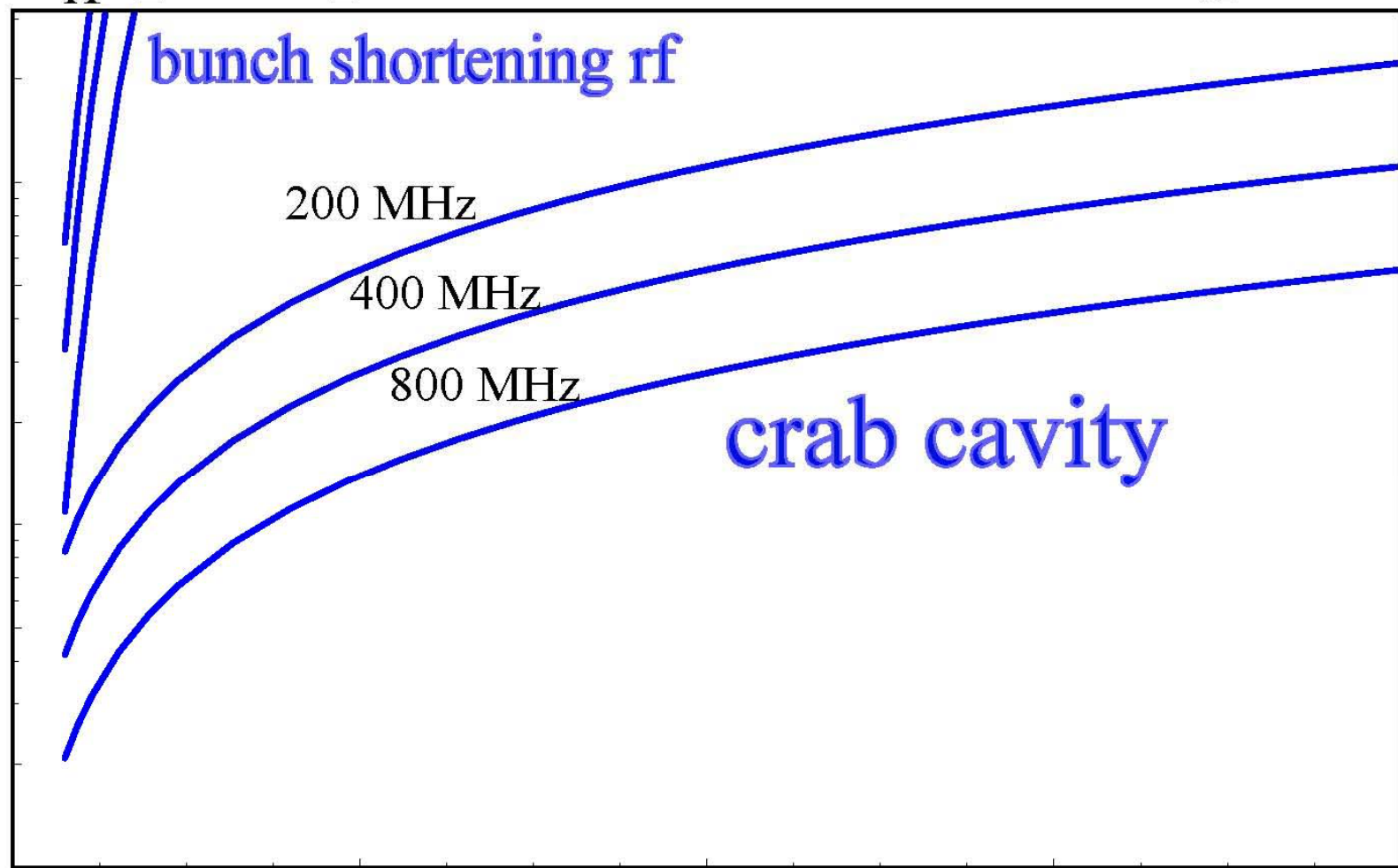
0.002

0.004

0.006

0.008

voltage required for Super-LHC  $\theta_c$  [rad]



crab cavity voltage for different  $\theta_c$ 's & rf frequencies

crossing angle	0.3 mrad	1 mrad	8 mrad
800 MHz	2.1 MV	7.0 MV	56 MV
400 MHz	4.2 MV	13.9 MV	111 MV
200 MHz	8.4 MV	27.9 MV	223 MV

\*800 MHz would be too high for nominal LHC bunch length

# tolerance on R22

$$\Delta\theta_c(z) \cong \frac{R_{22}}{R_{12}} \left( \frac{\theta_c}{2} z \right)$$

*z-dependent additional crossing angle*

$$\frac{\Delta\theta_c(2\sigma_z)\sigma_z}{2\sigma_x^*} = \frac{R_{22}}{R_{12}} \frac{\theta_c \sigma_z^2}{\sigma_x^*} \ll 1$$

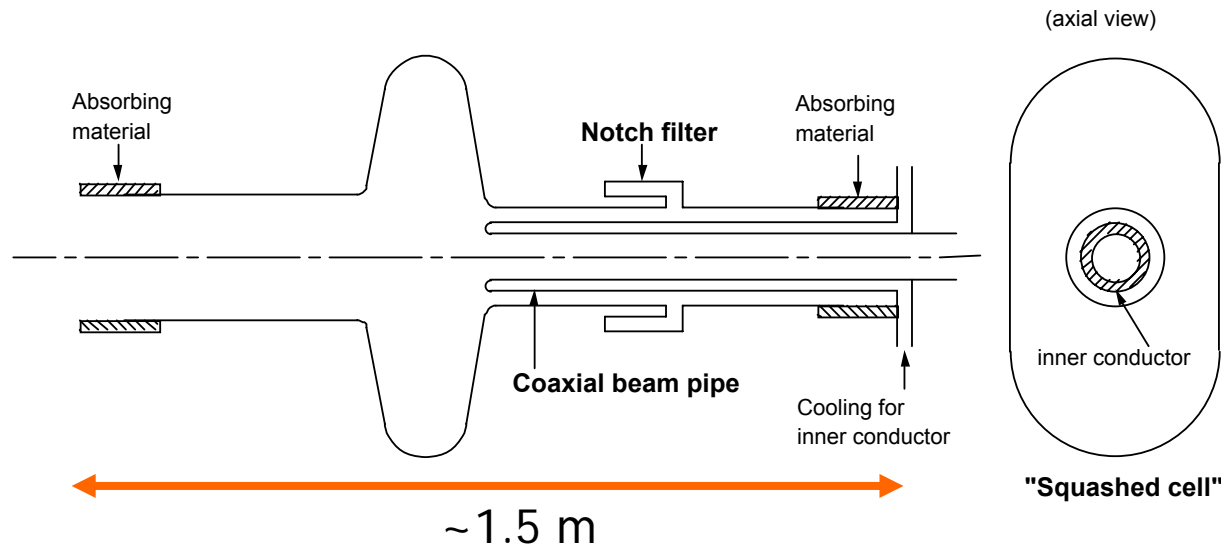
*corresponding Piwinski angle should be small*

$$R_{22} \ll \frac{R_{12} \sigma_x^*}{\theta_c \sigma_z^2} \approx 60 \mid \quad \text{not a problem}$$

[for  $\theta_c=1$  mrad,  $\sigma_x=12$  mm,  $R_{12}=30$  m,  $\sigma_z=7.55$  cm]

# KEKB crab cavity

- Squashed cell operating in TM<sub>2</sub>-1-0 (x-y-z)
- Coaxial coupler is used as a beam pipe
- Designed for B-factories (1 ~ 2A)



**Squashed Crab cavity for B-factories**

(K. Akai et al., Proc. B-factories, SLAC-400 p.181 (1992).)

Courtesy K. Akai



# longitudinal space & crab frequency

longitudinal space required for crab cavities  
scales roughly linearly with crab voltage;  
desired crab voltage depends on rf frequency);  
achievable peak field also depends on rf  
frequency; 2 MV ~ 1.5 m, 20 MV ~ 15 m  
frequency must be compatible with bunch  
spacing; wavelength must be large compared  
with bunch length;

$$\Delta x'(z) \cong \frac{1}{R_{12}} \left( \frac{\theta_c}{2} z - \frac{\theta_c}{2} \frac{1}{6} \frac{c^2}{\omega_{rf}^2} z^3 \right) + \dots \quad \omega_{rf} < \frac{\sqrt{6}c}{2\sigma_z} \cong 2\pi(775 \text{ MHz})$$

400 MHz reasonable

# noise

- amplitude noise introduces small crossing angle; e.g., 1% jitter  $\rightarrow 1\% \theta_c / 2$  cross. angle – tolerance  $< 0.1\%$  jitter from emittance growth
  - phase noise causes beam-beam offset; tolerance on LHC IP offset random variation  $\Delta x_{\max} \sim 10$  nm, from emittance growth
- $\rightarrow$  tight tolerance on left-right crab phase and on crab-main-rf phase differences

$$\Delta \phi_{crab} \leq \frac{\Delta x_{\max} 4\pi}{\lambda_{rf} \theta_c}$$

$$\Delta \phi < 0.012^\circ \quad (\Delta t < 0.08 \text{ ps})$$

at  $\theta_c = 1$  mrad & 400 MHz

$$\Delta \phi < 0.04^\circ \quad (\Delta t < 0.28 \text{ ps})$$

at  $\theta_c = 0.3$  mrad & 400 MHz

# comparison of timing tolerance with others

	KEKB	Super-KEKB	ILC	Super-LHC
$\sigma_x^*$	100 $\mu\text{m}$	70 $\mu\text{m}$	0.24 $\mu\text{m}$	11 $\mu\text{m}$
$\theta_c$	+/- 11 mrad	+/-15 mrad	+/-5 mrad	+/- 0.5 mrad
$\Delta t$	6 ps	3 ps	<b>0.03 ps</b>	<b>0.08 ps</b>



**IP offset of  $0.2 \sigma_x^*$**

IP offset of  
 $0.001 \sigma_x^*$   
 $\sim 10 \text{ nm}$

*→ not more difficult than ILC crab cavity*

# *p* emittance growth due to random offsets

$$\frac{\Delta \varepsilon}{\Delta t} \approx \frac{n_{IP} 8\pi^2 (\Delta x)_{rms}^2 \xi_{HO}^2 f_{rev}}{\beta^*}$$

emittance growth  
from turn-by-turn  
random collision  
offsets  $\Delta x$

SuperLHC:  $\beta_{x,y}^* = 0.25$  m,  $n_{IP} = 2$ ,  $\xi_{HO} = 0.005$ ,  
 $\gamma = 7500$ ,  $\gamma \varepsilon = 3.75$   $\mu\text{m}$

requiring less than 10%/hr emittance growth

—————→  $\Delta x_{rms} < 8$  nm  $\sim 10^{-3} \sigma^*$

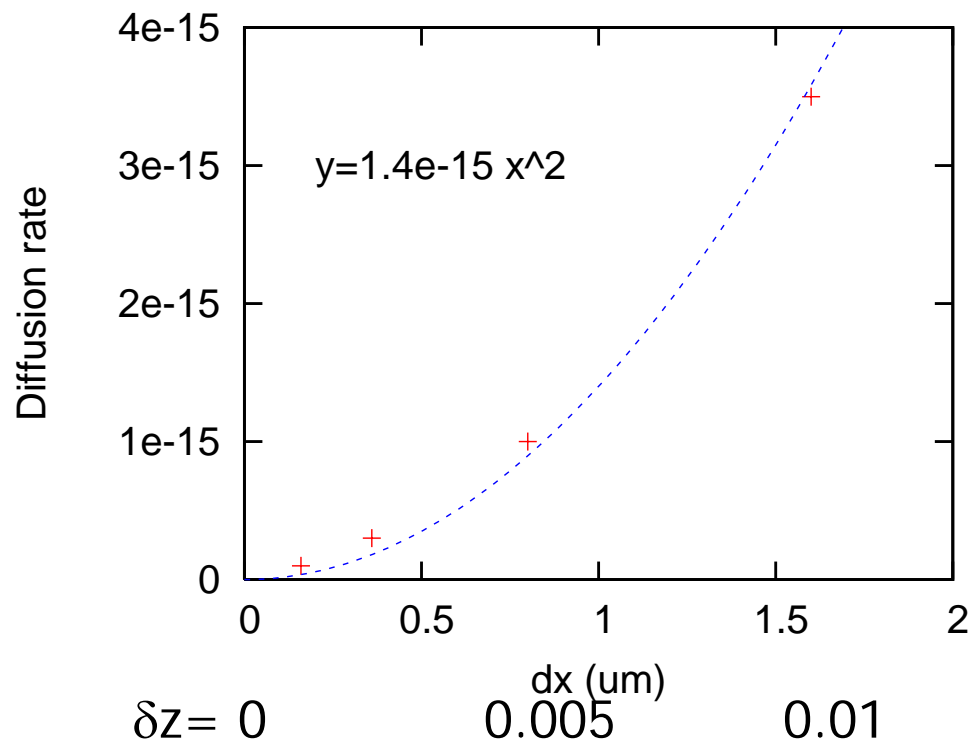
$$\Delta \phi < 0.008^\circ \text{ at } \theta_c = 1 \text{ mrad \& 400 MHz}$$

$$\Delta \phi < 0.027^\circ \text{ at } \theta_c = 0.3 \text{ mrad \& 400 MHz}$$

K. Ohmi, HHH-2004

# diffusion rate from strong-strong simulation with BBSS for nominal LHC

- $\sigma_x^2 = \sigma_{x0}^2 + Dt$      $t$ : turn
- $D \sim 1.4 \times 10^{-15} \Delta x [\mu\text{m}]^2$



## tolerance from Ohmi san's strong-strong simulation for nominal LHC

- For  $\Delta x = 1.6 \mu\text{m}$  ( $\delta\phi = 5$  degree) and  $\tau = 100$ ,  $D \sim 1.4 \times 10^{-15} \Delta x [\mu\text{m}]^2$ , where  $\sigma_x^2 = \sigma_{x0}^2 + Dt$ ,  $t$ : turn.
- Tolerance is  $\Delta x = 0.016 \mu\text{m}$ ,  $\Delta\phi = 0.05$  degree for  $\tau = 100$ , and  $\Delta x = 0.0016 \mu\text{m}$ ,  $0.005$  degree for  $\tau = 1$ , for luminosity life time  $\sim 1$  day

for  $300 \mu\text{rad}$  crossing angle  
and  $400 \text{ MHz}$

*slightly worse than my  
simple estimate!?*



# analytic theory of beam-beam diffusion

(T. Sen et al., PRL77, 1051 (1996)

M.P.Zorzano et al., EPAC2000)

- Diffusion rate due to offset noise. (round beam)

$$D(J_x) = \frac{(C\sigma|\delta x|)^2}{8 - 4/\tau} \sum_{k=0}^{\infty} \frac{\sinh \theta (2k+1)^2 G_k^2(a)}{\cosh \theta - \cos[2\pi(2k+1)v_x]}$$

$$G_k = \frac{\sqrt{a}}{\sigma} [U'_{k+1} + U'_k] + \frac{1}{\sqrt{a}\sigma} [(k+1)U_{k+1} - kU_k]$$

$$U_k(a) = \int_0^a \frac{1}{w} \left[ \delta_{0k} - (2 - \delta_{0k})(-1)^k e^{-w} I_k(w) \right] dw$$

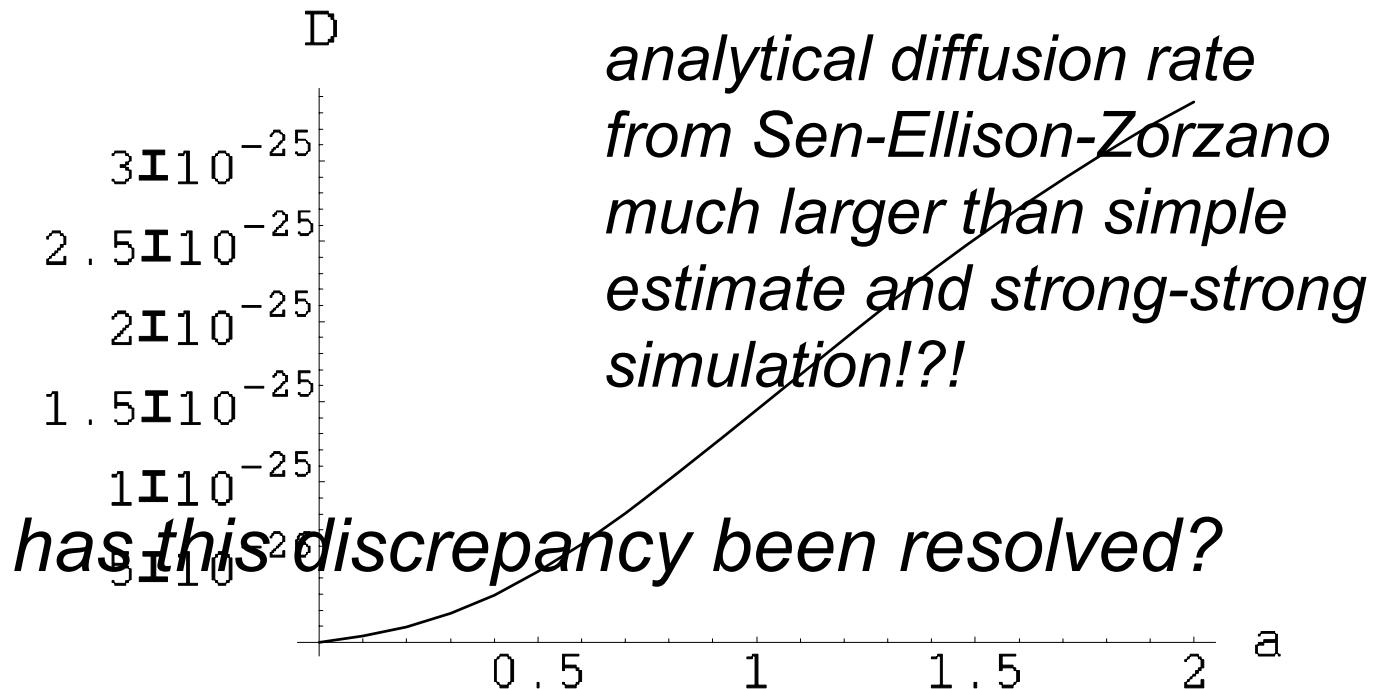
$$\theta = -\ln(1 - 1/\tau)$$

$$C = \frac{N_p r_p}{\gamma_p} \quad a = \frac{\beta^* J_x}{2\sigma^2}$$

K. Ohmi, HHH-2004

## comparison with the simulation

- $D(a=1)=\langle\Delta J^2\rangle=1.5\times 10^{-25}$  m<sup>2</sup>/turn
  - $D(\text{sim})=(\sigma-\sigma_0)^2/\beta^2=10^{-28}$  m<sup>2</sup>/turn
- “need to check”
- } 3 orders of magnitude discrepancy!



in addition to beam-beam offset, also the direct dipole kicks from random crab cavity phase jitter induce emittance growth

(J. Tuckmantel)

my estimate:

$$\frac{\Delta \varepsilon}{\Delta t} \approx \frac{f_{rev}}{\beta^*} \left( \frac{c \theta_c}{4\pi f_{crab}} \Delta \phi_{crab} \right)^2$$

example:

$$\beta^* \approx 0.25 \text{ m}, f_{crab} = 400 \text{ MHz.}, \Delta \phi_{crab} \approx 0.01 \text{ mrad}, \theta_c \approx 1 \text{ mrad}$$

$$\longrightarrow \frac{1}{\varepsilon} \frac{\Delta \varepsilon}{\Delta t} \approx 11\% / hr \quad \begin{matrix} (6 \times 10^{-4} \text{ }^\circ) \\ \sim 0.004 \text{ ps!} \end{matrix}$$

this effect likely requires transverse feedback, head-tail damping, or other scheme to suppress the dipole motion, or it can eliminate the idea altogether

impedance of crab cavities

transverse impedance is an issue  
due to large beta function

rise time due to 1 crab cavity

=

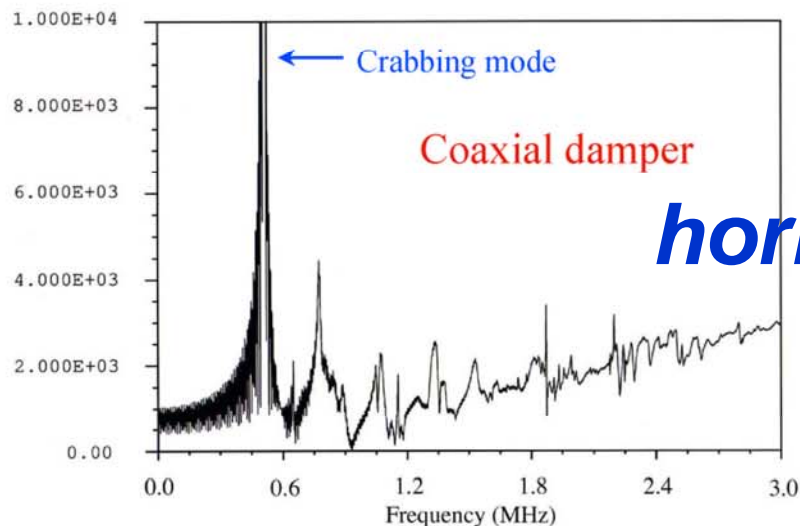
rise time from ~10 normal rf cavities  
with the same voltage

K. Akai

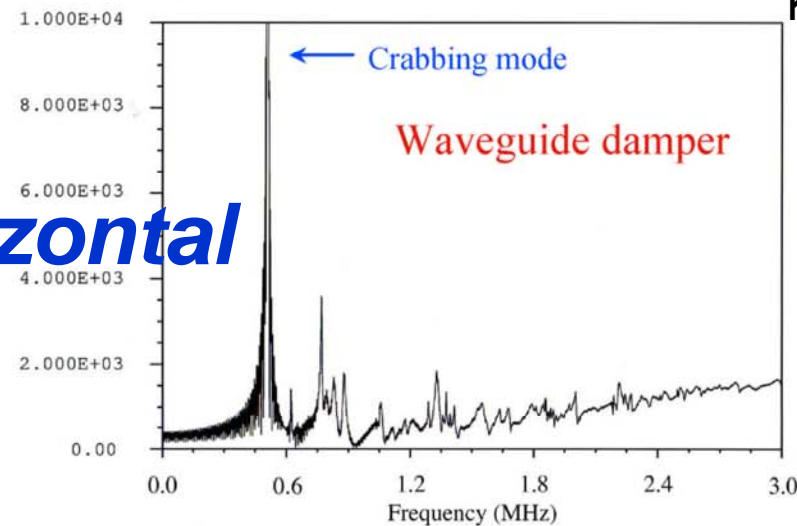
# Impedance of Super-KEKB Crab Cavity Design

K. Akai

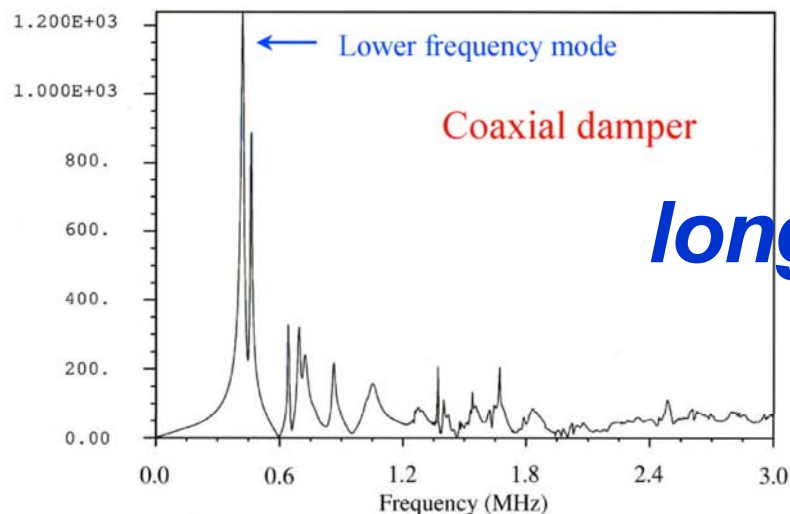
Impedance ( $k\Omega/m$ )



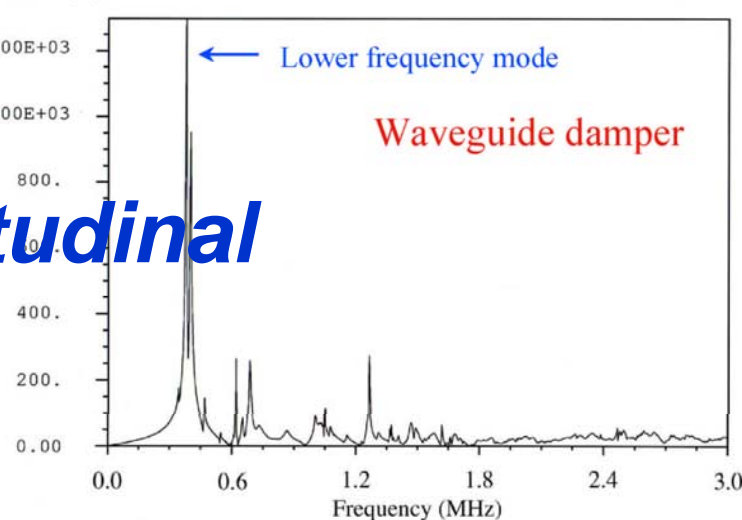
Impedance ( $k\Omega/m$ )



Impedance ( $\Omega$ )



Impedance ( $\Omega$ )



## merits of crab cavities

- practical demonstration at KEKB in early 2006
- avoids geometric luminosity loss, allowing for large crossing angles (no long-range beam-beam effect)
- potential of boosting the beam-beam tune shift (factor 2-3 predicted for KEKB)

## challenges & proposed plans

- design & prototype of Super-LHC crab cavity (Cornell and LBNL are interested)
- demonstration that noise-induced emittance growth is acceptable for hadron colliders (installation & experiment at RHIC?)